Charged Colloidal Structures in Plasmas

Sergey V. Vladimirov and Neil F. Cramer

Department of Theoretical Physics
School of Physics, The University of Sydney, NSW 2006, Australia

Abstract. Recent advances in the study of collective effects and particle motions in colloidal plasma crystals, ordered structures that form in the cathode sheath region of low temperature gaseous discharges, are presented. Plasma collective processes influence the arrangements and vibrations of colloidal "dust" particles in these Coulomb lattice structures. One important effect is the wake potential formation due to ion flow, leading to the vertical alignment of colloidal particles in experiments. Another effect is the influence of the plasma potential on the charge of the particles and their equilibrium positions. The oscillations of colloidal particles in the lattices are strongly affected by collective processes in the ambient plasma, in particular by the wake. Modes associated with vertical vibrations of colloidal particles are identified, and their dispersion characteristics are discussed.

INTRODUCTION

Colloidal "dusty" plasmas are plasmas containing small particles ranging in size from nanometres to micrometres. Physics of dusty plasmas has recently attracted considerable interest connected, in particular, with an increasing worldwide effort to model DC, RF, and microwave plasma discharge devices used in plasma-assisted materials processing. Dusty plasmas are also common in a variety of low-temperature plasmas in space environments, such as the lower ionosphere of the Earth, planetary atmospheres, asteroid zones, nebulas, and cometary tails. Another example of recent interest is the low temperature edge plasma physics in nuclear fusion devices, where dust grains emitted from walls may strongly influence anomalous transport properties.

An exciting area of the most recent research is the plasma-dust crystal formation [1]. The macroscopic lattices, named "dust-plasma crystals", are made of highly (negatively) charged particulates of micrometer size levitated in the sheath region above a horizontal negatively biased electrode. Although the dust-plasma crystal systems are generally three-dimensional, in the most experiments the charged particles are located just in few layers above the horizontal electrode, where gravity is balanced by the sheath electric field. These layers act mainly as two-dimensional systems, with limited out-of-plane particle motion [1]. On the other hand, it was demonstrated that aligning of dust grains from different layers in the vertical plane is strongly connected with plasma collective mechanisms, namely, with the presence of super-sonic ions flowing towards the negative electrode. It was shown [2] that in otherwise uniform plasma, the flow leaves a polarized oscillating wake potential behind a stationary dust particle, with ions focussing to make the plasma potential positive in a local region. This positive region attracts negative particles, promoting the vertical alignment clearly observed experimentally [1].

Here, recent advances in the study of collective effects and particle motions in dusty plasmas are presented. In particular, influence of plasma collective processes on arrangements and vibrations of dust particles in the crystall-like Coulomb structures is considered. Note that the wake potential formation which leads to the vertical alignment of dust grains in the experiments, and, as a consequence, to quasi-two-dimensional features of the structures (i.e., hexagonal arrangements) when the number of dust layers is not high (the three-dimensional lattices corresponding to minimum of the potential energy are body-centered-cubic or face-centered-cubic) can also affect the specific lattice modes propagating in such systems.

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14. ABSTRACT
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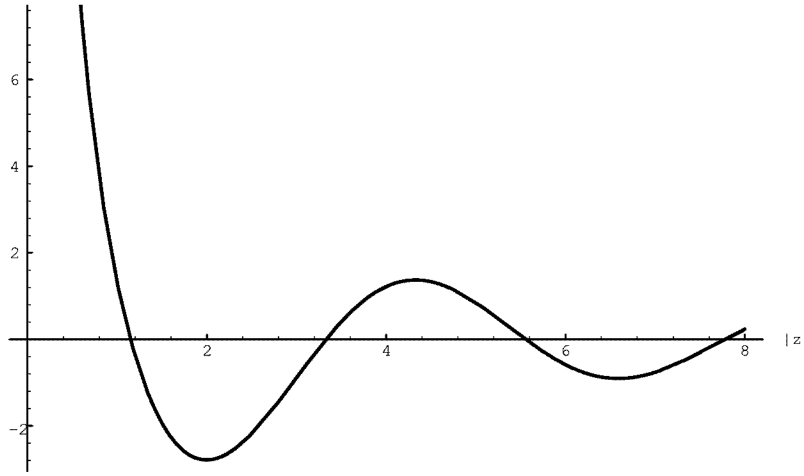

FIGURE 1. The wake potential on the line behind a test dust particle.

PLASMA WAKE

The drag force on a test particle in a plasma with finite flows necessarily includes collective effects. If the speed of the flow exceeds the velocity of ion oscillations in the flow, an oscillating stationary wake is formed behind the static test particle [2]; the effect is similar to the Cooper pairing of electrons in superconductors. We note that the collective mechanism can be responsible for the oscillatory potential in the direction parallel as well as perpendicular to the flow, and the attraction due to the wake potential can overcome the static Coulomb repulsion. The characteristic spacing in this case is of order Debye length λ_D in agreement with the experiments.

Consider the cylindrical geometry (ρ, φ, z); the plasma ions flow in the $-z$ direction with velocity v_{i0}; the test dust particle of the charge Q is placed on the position $(0,0)$. We calculate the potential behind the test particle downstream the flow within the wake cone: $|z| > \rho\sqrt{M^2-1}$, where $M = v_{i0}/v_s$ is the Mach number and $v_s = (T_e n_i/m_i n_e)^{1/2}$ is the sound velocity (T_e and n_e are the electron temperature and number density, and m_i and n_i are the ion mass and number density). The electrostatic potential of the static dust particle outside the Mach cone is

$$\Phi(\mathbf{r}) = \Phi_D(\mathbf{r}) = \frac{Q}{|\mathbf{r}|}\exp(-|\mathbf{r}|/\lambda_D), \tag{1}$$

where $\lambda_D = (T_e/4\pi n_e e^2)^{1/2}$, while inside the Mach cone the potential involves the collective effects caused by the oscillations in the ion flow:

$$\Phi(\mathbf{r}) = \Phi_W(\mathbf{r}) = Q\int \frac{d\mathbf{k}}{2\pi^2 k^2}\frac{k^2\lambda_D^2\omega_{\mathbf{k}}^2\exp(i\mathbf{k}\cdot\mathbf{r})}{(1+k^2\lambda_D^2)[(-k_z v_{i0}+i0)^2-\omega_{\mathbf{k}}^2]}. \tag{2}$$

Here, $\omega_{\mathbf{k}} = kv_s/(1+k^2\lambda_D^2)^{1/2}$ is the characteristic frequency of the oscillations in the flow. The potential (2) describes the strong resonant interaction between the oscillations in the ion flow and the test particulate when $|k_z v_{i0}|$ is close to $\omega_{\mathbf{k}}$.

For distances $k_\perp \rho \ll 1$ and $|z| > \lambda_D\sqrt{M^2-1}$, the main contribution to the stationary wake potential is given by

$$\Phi_W(\rho=0,z) \approx \frac{q_t}{|z|}\frac{2\cos(|z|/L_s)}{1-M^{-2}}, \tag{3}$$

where $L_s = \lambda_D\sqrt{M^2-1}$ is the effective length. This potential is presented on Fig. 1.

From Eq. (3), we can conclude that the wake potential is attractive for $\cos(|z|/L_s) < 0$. On the other hand, for distances $\rho > \lambda_D$ and $|z| > \lambda_D\sqrt{M^2-1}$ we find

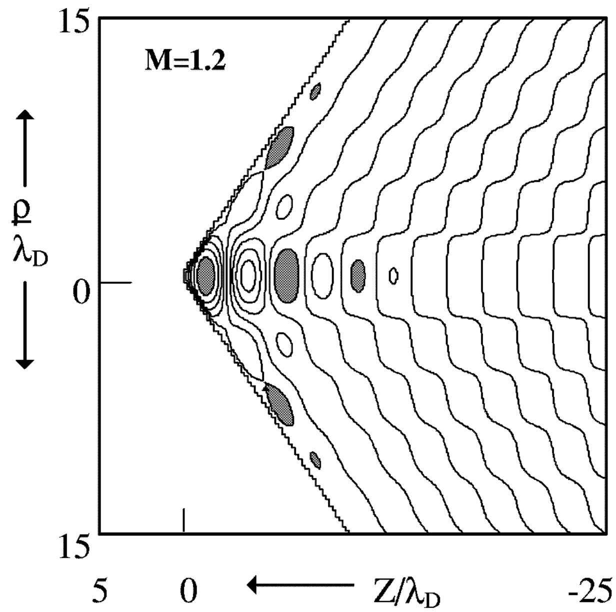

FIGURE 2. The wake potential in the Mach cone behind a test dust particle.

$$\Phi_W(\rho, z) \simeq \frac{Q}{1-M^{-2}}\sqrt{\frac{\lambda_D}{2\pi\rho}}\left\{\frac{\cos[(\pi/4)+(z_-/\lambda_D\sqrt{M^2-1})]}{z_-}\right.$$
$$\left.+\frac{\cos[(\pi/4)-(z_+/\lambda_D\sqrt{M^2-1})]}{z_+}\right\}, \tag{4}$$

where $z_\pm \equiv |z| \pm \rho\sqrt{M^2-1} > 0$. Because the oscillating potentials are proportional to the same dust particle charge Q as the static Debye potential, and contain no screening exponential, there are regions in space which correspond to the change of the effective potential sign and, hence, to the most probable positions of the particulates. We stress that these regions are not only on the line $\rho = 0$. The effective periodic spacing in the plane perpendicular to the flow is of order plasma Debye length. Behaviour of the potential in the Mach cone is presented on Fig. 2 (see also Ishihara and Vladimirov [2]). Direct experiments [3] confirmed the character of the wake potential.

The results of a self-consistent molecular dynamics (MD) three-dimensional (3D) simulation of the kinetics of plasma particles (electrons and ions) around a dust grain [4], taking into account the dust charging, are given on Fig. 3 where we present contour plots of the normalized ion density $n_/n_0i$ for three values of the speed of the ion flow (one is subsonic with $M^2 = 0.6$, and two supersonic, with $M^2 = 1.2$ and $M^2 = 2.4$). For better visualization, parts of the simulation volume where $n_i/n_0 < 1$ and $n_i/n_0 > 1$, respectively, are presented in the (greyscale) topograph style. A strong ion focus is formed at the distance of a fraction of the electron Debye length behind the dust grain. The maximum value of the density at the ion focus is almost independent of the flow velocity, whereas the characteristic distance of the ion focus from the dust grain increases with increasing flow velocity, being approximately equal to $0.5\lambda_D$ for $M^2 = 2.4$. This characteristic spacing corresponds to an ion focus effect in the near zone of the dust grain is a purely kinetic effect preceding the collective wake field formation. We note that the oscillating wake field which is formed in the wave zone behind the grain (see above) cannot form for the considered simulation time (half of the period of the ion oscillations).

Thus, the developed plasma collective effects can provide the attractive potential well on the line as well as in the plane perpendicular to the direction of the ion flow downstream the dust particle. Indeed, because the wake potential cannot change the sign of the effective potential at the distances less than the λ_D, the dust particulates are not expected to be arranged with the distances less than the Debye length. At the same time, the characteristic spacing of the polar radius-vector in the plane perpendicular to the flow is also of order λ_D. Therefore, we can expect the particulates on the equal distances λ_D on the periphery of the circle of the radius of order λ_D. This may correspond to polygon of order not higher than hexagon. Hexagonal structures were observed practically in all the experiments on dust crystallization [1].

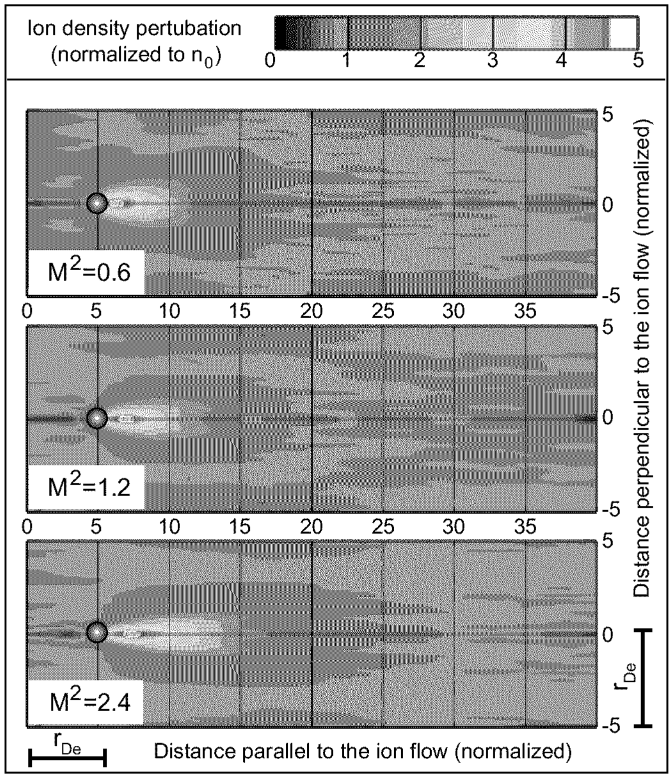

FIGURE 3. Contour plot of the ion density, showing ion focusing, for three different velocities of ion flow. The plot is presented in the greyscale topograph style; regions A correspond to ion densities below 1, and regions B correspond to ion densities above 1. The ions are focusing behind the grain thus forming the region with highly enhanced ion density. The distances are given in the units of $h_x \approx 1.077\mu$m used in the calculation; the physical distance corresponding to the electron Debye length r_{De} is presented.

DUST-LATTICE WAVES

The lattice waves in the dust crystal include motion of dust particles in horizontal direction as well as in the vertical direction. Consider vibrations of the two one-dimensional horizontal chains of particulates of equal mass M separated by the distance r_0 in the horizontal direction and d in the vertical direction [5], see Fig. 4.

The potential in the vertical direction acting on the lower particle due to the upper one is given by (3). The potential acting on the upper particle due to the lower particle is the simple Debye repulsive potential (1). The balance of forces in the vertical direction, in addition to the electrostatic Debye and the wake potential forces, includes the gravitational force $F = Mg$ as well as the sheath electrostatic force $F_{el} = QE(z)$ acting on the dust grains.

In equilibrium, since the interchain distance d is assumed to be small compared with the distance z_{01} of the lower chain from the electrode (as well as small compared with the width of the sheath), we can assume that the sheath electric field in the range of distances z_{01} to $z_{02} = z_{01} + d$ can be linearly approximated as $F_{el} - Mg = -\gamma(z - z_0)$, where γ is a constant and z_0 is the equilibrium position of a particle of mass M due to the forces Mg and F_{el} only. We stress that the actual equilibrium positions of particles in lower and upper chains are z_{01} and z_{02}, respectively. The equilibrium balance of the forces in the vertical direction acting on the lower chain and the upper chain can be written as

$$F_{el,1}(z_{01}) - Mg + F_1^0(z_{02} - z_{01}) = 0, \qquad F_{el,2}(z_{02}) - Mg + F_2^0(z_{02} - z_{01}) = 0, \tag{5}$$

where $F_{1,2}^0$ are the forces of interaction between the chains due to the potentials (3) and (1). Here, since we have $z_{02} = z_{01} + d$,

$$F_1^0(z_{02} - z_{01}) = Q\frac{d\Phi_1(|z|)}{d|z|}\bigg|_{|z|=d}, \quad F_2^0(z_{02} - z_{01}) = -\,Q\frac{d\Phi_2(|z|)}{d|z|}\bigg|_{|z|=d}. \tag{6}$$

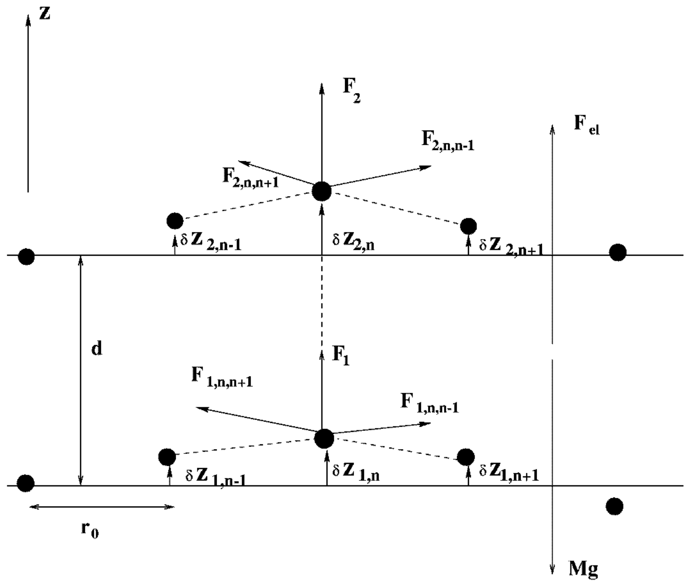

FIGURE 4. Forces acting on dust particles in two one-dimensional horizontal chains: the gravitational Mg and the sheath electric F_{el} external fields, as well as the wake F_1 and Debye F_2 interaction fields.

The equilibrium distance d can be found from equations (5): $F_2^0(d) - F_1^0(d) = \gamma d$.

By introducing small perturbations $\delta z_{i,n}$ of the equilibrium at z_{0i}, where $i = 1, 2$ for the lower and upper chains, respectively, including interactions with nearest neighbours in each chain, and substituting $\delta z_{i,n} = A_i \mathrm{exp}(-i\omega t + iknr_o)$ we obtain

$$\omega_1^2 = \frac{\gamma}{M} - \frac{4Q^2}{Mr_0^3}\mathrm{e}^{-r_0/\lambda_D}(1 + r_0/\lambda_D)\sin^2\frac{kr_0}{2}, \quad A_1 = A_2, \tag{7}$$

$$\omega_2^2 = \frac{\gamma + \gamma_1 - \gamma_2}{M} - \frac{4Q^2}{Mr_0^3}\mathrm{e}^{-r_0/\lambda_D}(1 + r_0/\lambda_D)\sin^2\frac{kr_0}{2}, \quad A_1 = A_2\frac{\gamma_1}{\gamma_2}, \tag{8}$$

where

$$\gamma_1 = Q\frac{d^2\Phi_1(|z|)}{d|z|^2}\bigg|_{|z|=d}, \quad \gamma_2 = -\,Q\frac{d^2\Phi_2(|z|)}{d|z|^2}\bigg|_{|z|=d}. \tag{9}$$

We see that for $k = 0$ the characteristic frequencies are given by $\omega_1^2 = \gamma/M$ and $\omega_2^2 = (\gamma - \gamma_1 - \gamma_2)/M$, and they decrease with growing wave number when $kr_0 \ll 1$.

To estimate the effective width of the electrode potential well γ, we employ the standard model of the sheath, which considers Boltzmann distributed electrons. For simplicity, we ignore the influence the dust grains may have on the field distribution in the sheath region. From Poisson's equation, integrated it once and linearizing, we obtain the effective width of the electrode potential well

$$\gamma = 4\pi e|Q|n_0\left[\exp\left(\frac{e\phi_0}{T_e}\right) - \left(1 - \frac{2e\phi_0}{T_e}\frac{v_s^2}{v_0^2}\right)^{-1/2}\right]. \tag{10}$$

Solutions of equations for the sheath potential ϕ_0 in the equilibrium position and for the equilibrium distance d in the vertical direction can be found numerically. Assuming $\lambda_D \approx 2 \times 10^{-2}$cm, $v_0^2/v_s^2 \approx 1.5$, $Q/e = 2 \times 10^4$, $M = 0.6 \times 10^{-9}$g, the equilibrium vertical distance is given by $d = 1.75\lambda_D$, and $\gamma_1/\gamma_2 \approx -25.3$. The characteristic frequencies of the two modes are approximately

$$f_1(k=0) = \frac{1}{2\pi}\sqrt{\frac{\gamma}{M}} \approx 21.3Hz, \quad f_2(k=0) = \frac{1}{2\pi}\sqrt{\frac{\gamma + \gamma_1 - \gamma_2}{M}} \approx 63.5Hz. \tag{11}$$

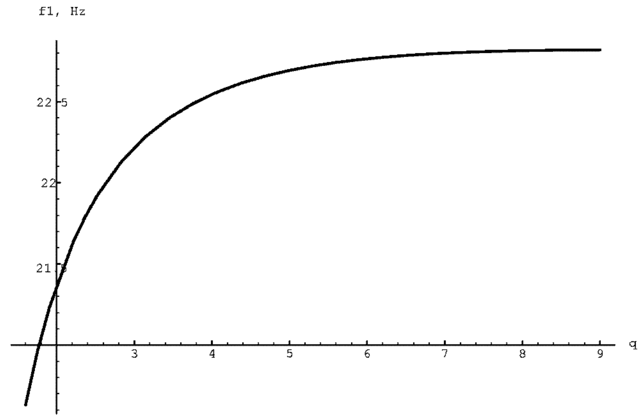

FIGURE 5. Dependence of frequency f_1 of the first mode (the amplitudes are the same and in phase for the vertically arranged grains in the first and in the second chains) vs dimensionless grain charge $q = |Q/e| \times 10^{-4}$.

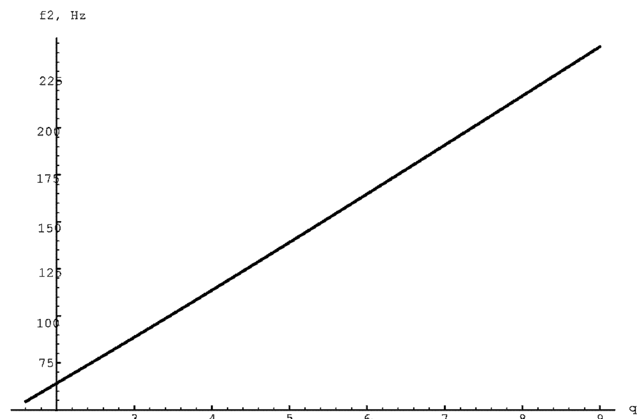

FIGURE 6. Dependence of frequency f_2 of the second mode (the amplitudes are different and in counter phase for the vertically arranged grains in the first and in the second chains) vs dimensionless charge $q = |Q/e| \times 10^{-4}$.

Their dependencies on Q are presented on Figs. 4 and 5 (see also Vladimirov, Shevchenko, and Cramer [5]).

Since the equilibrium distance d between the chains is almost independent of Q, the frequency f_2 is approximately directly proportional to Q. Note that the amplitude of dust grain oscillations in the lower chain for the second mode (when the grains oscillate with opposite phases) is much smaller than the amplitude of the oscillations in the upper chain, $|A_1/A_2| = |\gamma_1/\gamma_2| \approx 25.3$. For the first mode, the amplitudes are the same for the upper and the lower particles.

Thus the vertical oscillations of the two one-dimensional chains of dust grains levitating in the sheath field of a horizontal negatively biased electrode can give rise to two specific low-frequency modes which are characterized by an inverse optic-mode-like dispersion when the wave lengths far exceed the intergrain distance. Excitation of these modes can be responsible for phase transitions in the system. Note also that the second mode may be especially useful for diagnostic purposes because its dispersion is almost directly proportional to the dust grain charge Q which in turn is a function of plasma parameters.

The above model dealt with dust grains of a constant charge. However, it is well known that the charge of dust particles, appearing as a result of electron and ion current onto the grain surfaces, strongly depends on the parameters of the surrounding plasma. Thus, it was demonstrated [6] that the dependence of the dust particle charge on the sheath parameters has an important effect on the oscillations and equlibrium of dust grains in the vertical plane, leading to a disruption of the equilibrium position of the particle and a corresponding transition to a different vertical arrangement. The dependences of the Mach number of the ion flow and the dust grain charge, found using the charging equation, on the distance from the electrode are presented in Fig. 7.

If the electrode potential becomes even more negative, and a dust grain is very close to the electrode, its charge can become zero and, possibly, positive; for real conditions this only means that the particle cannot levitate at this distance. For a particle levitating in the sheath field the dependence of the charge as a function of its size is shown in Fig. 8. Note that there are no equilibrium solutions for $a > a_{\max} = 3.75\mu$m. However, by considering dust vertical vibrations [6], the disruption of the equlibrium happens in fact for a slightly lesser

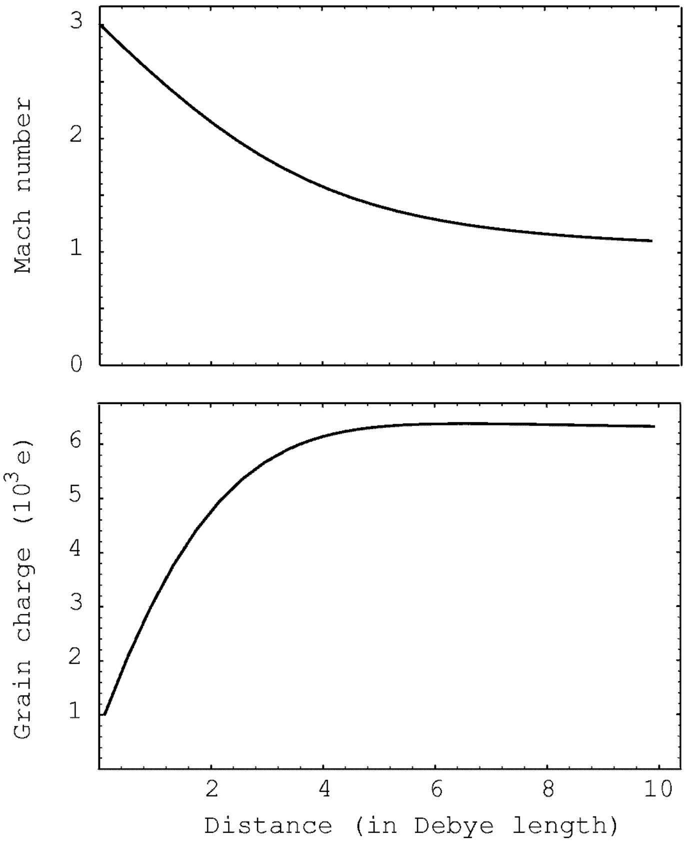

FIGURE 7. The dependence of the Mach number of the ion flow $M(z) \equiv v_i(z)/v_s$ and the charge of the dust grain $q(z) = -(Q(z)/e) \times 10^{-4}$ on the distance $h = z/\lambda_{D0}$ from the electrode. Here we have $M_0 = v_0/v_s = 1$, $\lambda_{D0} = 3 \times 10^{-2}$cm, $T_e = 1$eV, and $m_i/m_e = 40 \times 10^3$, and $a = 0.35 \times 10^{-3}$cm.

size $a_{\rm cr} < a_{\rm max}$.

It is also instructive to find the total potential energy, relative to the electrode position, of a single dust particle of given size at the position z in the sheath electric field; its dependence on the distance from the electrode is shown in Fig. 9. For comparison, we also plot here the energy in the case of a constant Q placed at the same equilibrium (or marginal equlibrium) position. We see that the potential always has a minimum for the case of Q =const, but in the case of a variable charge there can be a maximum and a minimumII. The minimum (the stable equilibrium) disappears if $a > a_{\rm cr}$. For the parameters considered here, for $a = a_{\rm cr} = 0.372 \times 10^{-3}$cm, the minimum disappears. This is close to the critical radius observed in experiments [7].

Thus for the collisionless sheath, for a less than the critical radius, there is an unstable equilibrium position deep inside the sheath, and a stable equilibrium position closer to the presheath. For a greater than the critical radius there is no equilibrium position. Vertical oscillations about the stable equilibrium may develop high amplitudes. This may lead to a fall of the oscillating grain onto the electrode when the potential barrier is overcome. Such a disruption of the dust motion has been observed experimentally [7].

To conclude, we have demonstrated that the charge, position, and spectrum of vertical oscillations of dust grains levitating in a low-temperature gas-discharge plasma strongly depend on the parameters of the plasmal. The dependence of the particle charge on the plasma potential is crucial for the characteristics of the mode associated with vertical vibrations as well as for the equilibrium of the dust particles. Large amplitude vertical oscillations of the dust grains, with frequencies derived here, may be responsible for experimentally observed disruptions of the equilibrium of the dust crystal as well as with numerically demonstrated phase transitions associated with vertical rearrangements of the grains.

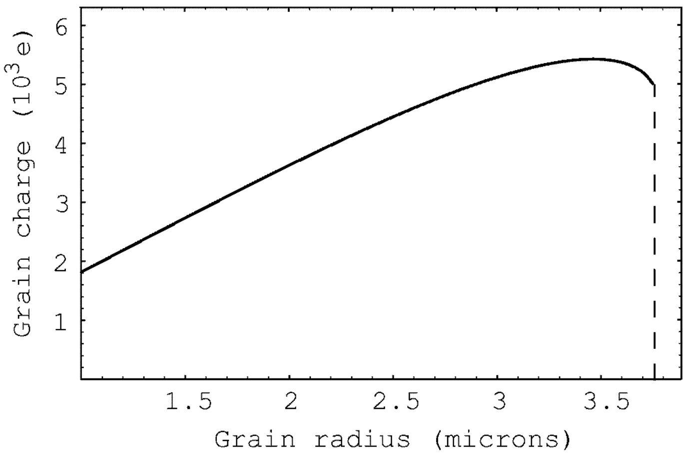

FIGURE 8. The dependence of the charge $q = -(Q/e) \times 10^{-3}$ of the dust grain, levitating in the sheath electric field, on its size. Here we have $M_0 = v_0/v_s = 1$, $\lambda_{D0} = 3 \times 10^{-2}$cm, $T_e = 1$eV, $m_i/m_e = 40 \times 10^3$, $\rho = 1$g/cm^3, and $a_{\max} = 0.375 \times 10^{-3}$cm.

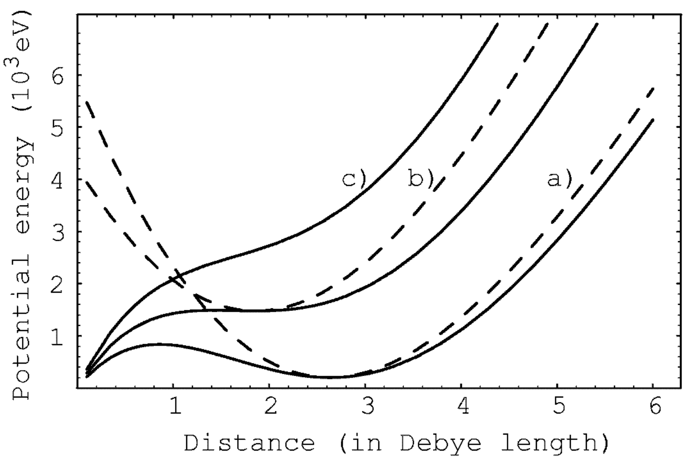

FIGURE 9. The total interaction energy U_{tot} as a function of the distance $h = z/\lambda_D$ from the electrode for the different sizes of a dust particle: *a)* $a = 0.35 \times 10^{-3}$cm; *b)* $a = a_{\text{cr}} = 0.372 \times 10^{-3}$cm; and *c)* $a = 0.4 \times 10^{-3}$cm. The dashed lines correspond to the case of a charge constant at the equilibrium (or marginal equilibrium) position: a) $Q = -5.42e \times 10^3$ and b) $Q = -4.93e \times 10^3$.

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REFERENCES

1. H.M. Thomas and G.E. Morfill, Nature **379**, 806 (1996).
2. S.V. Vladimirov and M. Nambu, Phys. Rev. E **52**, R2172 (1995); S.V. Vladimirov and O. Ishihara, Phys. Plasmas **3**, 444 (1996); O. Ishihara and S.V. Vladimirov, Phys. Plasmas **4**, 69 (1997).
3. K. Takahashi, T. Oishi, K. Shimonai, Y. Hayashi, and S. Nishino, Phys. Rev. E 58, 7805 (1998).
4. S.A. Maiorov, S.V. Vladimirov, and N.F. Cramer, submitted for publication.
5. S.V. Vladimirov, P.V. Shevchenko, and N.F. Cramer, Phys. Rev. E, **56**, R74 (1997); Phys. Plasmas **5**, 4 (1998).
6. S.V. Vladimirov, N.F. Cramer, and P.V. Shechenko, Phys. Rev. E **60**, 7369 (1999).
7. S. Nunomura, T. Misawa, N. Ohno, and S. Takamura, Phys. Rev. Lett. **83**, 1970 (1999).